

## MentiMATH - The concept of derivatives

Application and short description: This Mentimeter template elaborates with the concept of derivatives and the intended target group is students in 11th grade (US Curriculum). The template is designed foremost as an introduction but can also be used in the middle or as a form of reinforcement at the end of a subject block surrounding derivatives.

### Target group

- Students in the 11th grade (US Curriculum).

### Estimated time

- Approximately 40-90 minutes for **all slides**.
- Approximately 1-5 min for **each slide**. (The amount of time each slide takes could depend on several variables, for example the students' mathematical knowledge, the teacher's need for clarification within each slide and what work has been done in advance)

### Learning goals

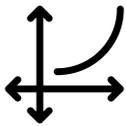
- Understand the *limit definition of derivatives*.
- Get a visual understanding of derivatives as *the slope of the tangent line at a point*.

**Theoretical background:** The concept of a derivative is an essential part of calculus, and has enormous practical applications. However, research has shown that it is very common among students to not fully grasp the concept. It is theorized that this is due in part because of a lack of prerequisite knowledge. Asiala et al. (1997) have investigated students' conceptions of derivatives. They stress the importance that students have a rich conception of a *function*, including the  $f(x)$ -notation. Other things they mention as important prerequisites are the graphical representation of a slope of a line in a graph, and points in a coordinate system. Therefore, the concept of a *slope* gets a lot of attention in the early parts of the template in order to fill potential gaps of knowledge. Also Asiala et al. (1997) state it is important for students to see: "the relationship between the derivative of a function at a point and the slope of the line tangent to the graph of the function at that point.". This visual representation is also a core part of the presentation.

**How to use the template:** Here follows some ideas on how the slides are intended to be used but feel free to use the template however you like. Included in each cell is the purpose and other information that could be of use for you as a teacher. A general idea regarding all questions is that the students should work after the model Think-Pair-Share (TPS). When implementing TPS the students should first answer the question individually, then discuss in pairs (or groups of three) followed by a classroom discussion led by the teacher. As a concluding remark, the content in the slides are designed primarily for developing students' understanding of *mathematical concepts* rather than the procedural 'know how'. Therefore, discussions and questions from the students is a central part of using the presentation where your expertise as a teacher is important to clarify misconceptions and lead the classroom discussions.

**Important keyboard shortcuts:** To get the most out of presenting with Mentimeter, there are a few keyboard shortcuts you need to know.

- Toggle between hiding and showing results: **H**. It is important to not show results before everyone has had a chance to give their response since this can influence other students' thinking.
- Countdown timer: **1** (60 seconds), **8** (30 seconds) **9** (10 seconds), **0** (stop countdown). It is good to be flexible and to start a timer (either, 60, 30 or 10 seconds) when you feel that the majority of the group is ready to move on from a discussion.
- Show correct answers: **ENTER**. Wait until all students have answered a multiple choice question before you show the correct answer(s).



## The concept of a derivative

Determining the instantaneous rate of change

### Slide 1 - Introduction.

Introduce the theme of the lecture and present the learning goals. For example you could mention that derivatives is an essential part of calculus that have many practical implications.

## Introduction

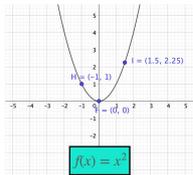
You are going to work with the limit definition of the derivative. On every problem you submit your answers individually. After that you discuss in pairs (or in a group of three), then the whole class together.

All answers are anonymous except for the quiz in the end where you type in your name.

To be able to answer the question, go to [www.mentimeter.com](http://www.mentimeter.com), then plug in the code you can see at the top of this slide.

### Slide 2 - General information.

Inform the students regarding the TPS methodology, that their responses are anonymous and how they can access the interactive presentation.

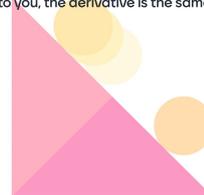


Criteria for success: Understand the limit definition of the derivative and be able to use the derivative to calculate the slope at different points.

### Slide 3 - Criteria for success.

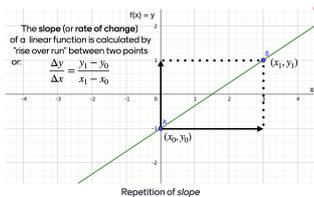
Introduce a 'criteria for success' for the lesson with a problem that will be revisited and solved at the end of the lesson. The idea here is to make the learning visible, so the students can see for themselves that they've actually learned something.

According to you, the derivative is the same thing as the...



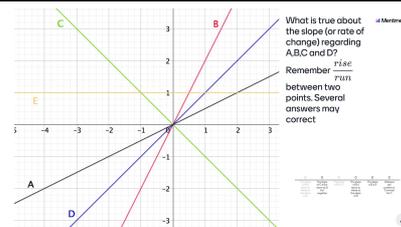
### Slide 4 - According to the students, a derivative is...

Let the students individually write down how they see the concept of a derivative. This can help clarify potential misconceptions.



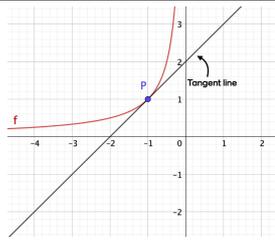
### Slide 5 - Prerequisites for this lesson. [Content slide].

Repetition of the concept of a *slope*. Research has shown that this is an important prerequisite to be able to understand derivatives.



### Slide 6 - Understanding slope.

This slide presents an opportunity for thinking about what a slope really means in mathematical terms, and you as a teacher get a chance to see if your students actually feel comfortable calculating the slope.

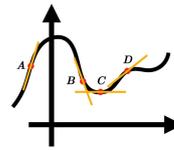


How could we know the slope of a point on a non-linear function? Draw a tangent line! What is the slope of the function  $f$  at the point  $P$ ?



**Slide 7 - The slope of points in nonlinear functions.**

Introduce what a *tangent line* is. This is a key concept in order to get a visual understanding of the derivative.

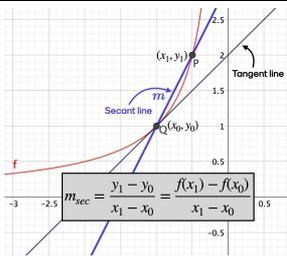


What is the true about the slope of the graph at A, B, C and D? Several answers may be correct.



**Slide 8 - The slope at points in nonlinear functions.**

This slide reinforces the concept of tangent lines, and how a tangent line can be used to think about the slope at a point in a nonlinear function.



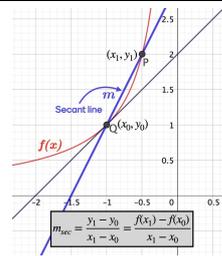
A secant line passes through two points (P and Q in this case) of a curved line. What is the slope of the secant line?

$$m_{sec} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$



**Slide 9 - The slope of a secant line.**

Introduce the concept of a secant line. This is a key concept for understanding the derivative.

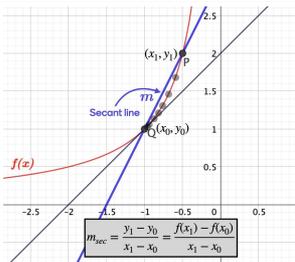


$f(x) = \frac{1}{x}$ . Look at the alternatives. What is the correct calculation of  $m_{sec}$ ?



**Slide 10 - Calculating the slope of the secant line using only the x-value and the function.**

This is a key slide! Here the students need to understand how the secant line can be calculated using only  $x_0$  and  $x_1$ , and the function (without direct information of  $y_0$  and  $y_1$ ). It is very important that all students grasp the content of this slide in order to be able to grasp the limit definition of the derivative.



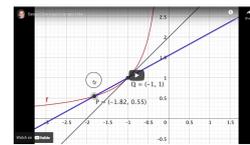
Imagine that  $x_1 \rightarrow x_0$ . Then P will approach Q along the graph of the function  $f(x)$ . When P approaches Q, what is true for secant line?

$$m_{sec} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$



**Slide 11 - Decreasing the distance between the points of the secant line.**

This is a key slide! The idea here is to make the students think about what will happen when the distance between the two points on the secant line becomes really small.



The slope of the secant line through P and Q will gradually approach the slope of the tangent line through P as the distance between P and Q decrease

**Slide 12 - The secant line and tangent line becomes the same [Content slide].**

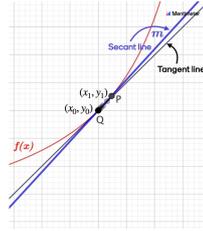
Give a visual representation of what happens when the distance between the two points on the secant line becomes really small.

So we had the equation for the secant line:

$$m_{\text{sec}} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

In the limit when  $x_1 \rightarrow x_0$  we get the equation for the tangent line:

$$m_{\text{tan}} = \lim_{x_1 \rightarrow x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$



### Slide 13 - Going from the secant line to the tangent line. [Content slide].

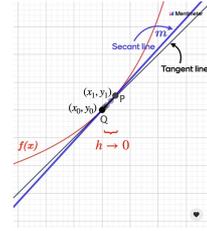
Go from the definition of the secant line to an expression for the tangent line. This is key for understanding the derivative as the slope of the tangent line at a point in a graph.

Now if we let  $x_1 - x_0 = h$ , then  $h \rightarrow 0$  as  $x_1 \rightarrow x_0$ . If we rewrite  $x_1$  as  $h + x_0$  we can rewrite the equation as:

$$m_{\text{tan}} = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

When the limit exists, this is the derivative of  $f(x)$  at  $x_0$ . So the derivative at a point (Q in this case) is the same thing as the slope of the tangent line at that point. If we replace  $x_0$  from our example with any  $x$ , we get the definition of the derivative of a function  $f(x)$ :

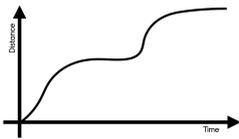
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$



### Slide 14 - Introduce the limit definition of a derivative [Content slide].

This is a key slide! Go from the specific case with two points (Q and P) to the general case. Talk about the concept of a *limit*. Here it can be a good idea to let the students write down the limit definition of the derivative.

Here you can see the graph of a function representing the movement of a car. Pin where the derivative reaches its highest value!



### Slide 15 - Derivative in a practical situation.

Here the students will need to use their graphical understanding of the derivative as the slope of the tangent line at different points along the graph. Also, they will start to think about practical implications of the derivative.

Below is a calculation of the derivative of the function  $f(x) = x^2$ , using the limit definition of the derivative. Pin where you get lost in the calculation, or pin the green box if you understand every step.

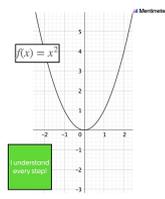
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x^2 + 2xh + h^2) - x^2}{h}$$

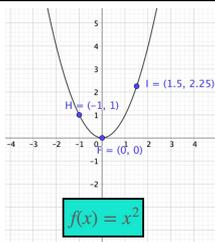
$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = 2x + h$$

$$h \rightarrow 0 \Rightarrow f'(x) = 2x$$



### Slide 16 - Using the limit definition.

This is a key slide! Here you as a teacher can get a sense of the current level of understanding. Be sure to spend a lot of time giving clarifications what the expression for the derivative ( $2x$ ) really means. This is essential information for the next slide.



Criteria for success revisited: Using the derivative of the function,  $f'(x) = 2x$ , what is the derivative at points H, F and F?

### Slide 17 - Criteria for success revisited.

Return to the problem that was introduced in the beginning of the lecture to make the students realize they've actually learnt something.

What is still unclear about the derivative of functions?

0 questions  
0 upvotes

### Slide 18 - Q & A.

Let the students individually write down what they still feel is unclear about derivatives.

# Quiz!

## Slide 16 - 24 - Quiz competition!

Instruct the students to enter their real names. Let the students compete in a fun way while you as a teacher can get a sense of the individual level of knowledge. This information could be used to give support for students in need of extra guidance.

### References:

Asiala, M., Cottrill, J., Dubinsky, E., & Schwingendorf, K. E. (1997). The development of students' graphical understanding of the derivative. *The Journal of Mathematical Behavior*, 16(4), 399-431.