

MentiMATH - The concept of derivatives

Application and short description: This Mentimeter template could be used as a continuation after applying the mentimeter-module '*The concept of a function*' but also works on its own. The template elaborates how functions could be visualized and understood with a graphical representation. The template is designed foremost as an introduction but can also be used in the middle or as a form of reinforcement at the end of a subject block surrounding linear functions..

Target group

- Students in the 8th grade (US Curriculum).

Estimated time

- Approximately 40-90 minutes for **all slides**.
- Approximately 1-5 min for **each slide**. (The amount of time each slide takes could depend on several variables, for example the students' mathematical knowledge, the teacher's need for clarification within each slide and what work has been done in advance)

Learning goals

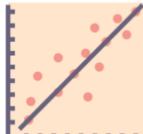
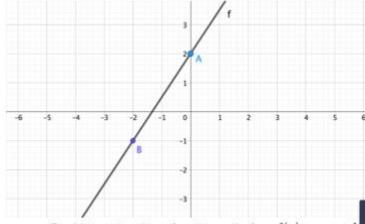
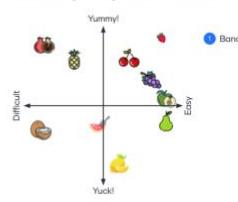
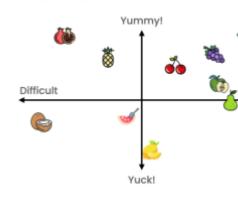
- Be able to tell which option is most affordable when choosing between two price-alternatives for traveling by public transport.
- The connection between a linear function and its graph
- See if a function represented by a graph increase or decrease
- Be able to express a function with its graph

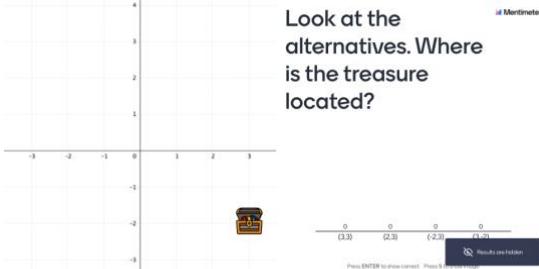
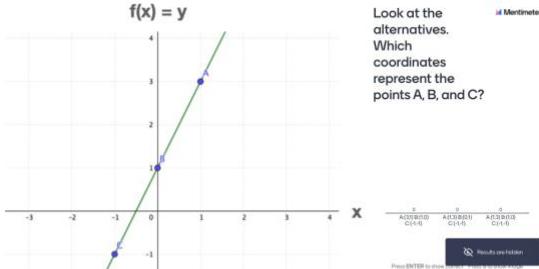
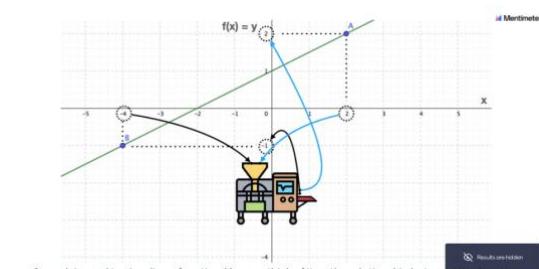
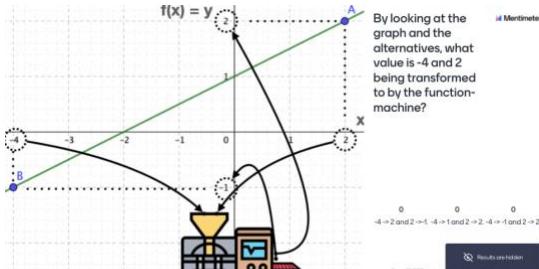
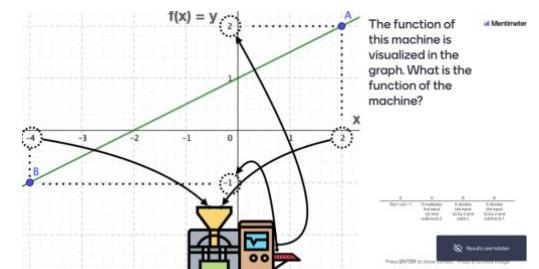
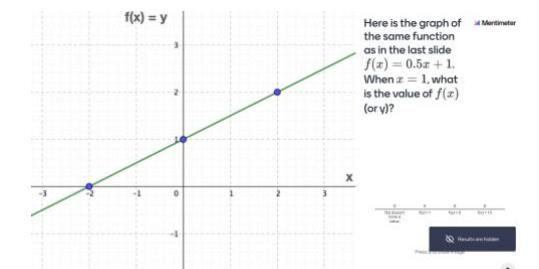
Theoretical background: A *function* is defined as a threshold concept. This means that it is *transformative* - once learnt it opens up new ways of conceiving a concept and it will be impossible to forget. Not grasping the concept of a function will hinder the learner to get a rich mathematical understanding later down the line. Students sometimes make conceptual mistakes when interpreting graphic representations of functions. To truly understand a graph correctly the students could benefit from a thoughtful design of the learning software in combination with a deeper mathematical knowledge. This template could therefore facilitate the understanding of functions represented by graphs but requires some mathematical knowledge in order to link the learning possibilities with the template's content and desired outcomes.

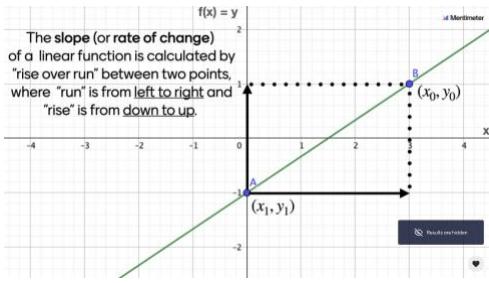
How to use the template: Here follows some ideas on how the slides are intended to be used but feel free to use the template however you like. Included in each cell is the purpose and other information that could be of use for you as a teacher. A general idea regarding all questions is that the students should work after the model Think-Pair-Share (TPS). When implementing TPS the students should first answer the question individually, then discuss in pairs (or groups of three) followed by a classroom discussion led by the teacher. As a concluding remark, the content in the slides are designed primarily for developing students' understanding of *mathematical concepts* rather than the procedural 'know how'. Therefore, discussions and questions from the students is a central part of using the presentation where your expertise as a teacher is important to clarify misconceptions and lead the classroom discussion.

Important keyboard shortcuts: To get the most out of presenting with Mentimeter, there are a few keyboard shortcuts you need to know.

- Toggle between hiding and showing results: **H**. It is important to not show results before everyone has had a chance to give their response since this can influence other students' thinking.
- Countdown timer: **1** (60 seconds), **8** (30 seconds) **9** (10 seconds), **0** (stop countdown). It is good to be flexible and to start a timer (either, 60, 30 or 10 seconds) when you feel that the majority of the group is ready to move on from a discussion.
- Show correct answers: **ENTER**. Wait until all students have answered a multiple choice question before you show the correct answer(s).

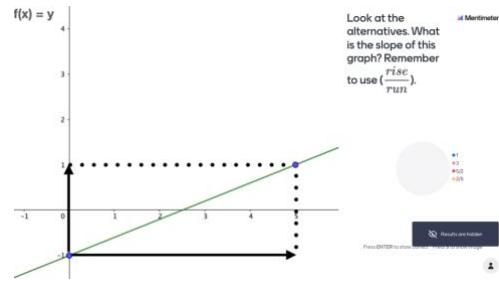
<p>Go to www.menti.com and use the code 5478 1004</p> <p></p>  <p>The graph of a function Visualizing relationships</p> <p></p> <p>Slide 1 - The graph of a function.</p> <p>Introduce the theme of the lecture. For example you could mention that the graphical representation of a function is very central to mathematics.</p>	<p>Go to www.menti.com and use the code 5478 1004</p> <p></p> <p>Introduction</p> <p>You are going to work with the graph of linear functions. On every problem you submit your answers individually. After that you discuss in pairs (or in a group of three), then the whole class together.</p> <p>All answers are anonymous except for the quiz in the end where you type in your name.</p> <p>To be able to answer the question, go to www.menti.com, then plug in the code you can see at the top of this slide.</p> <p></p> <p>Slide 2 - General information.</p> <p>Inform the students regarding the TPS methodology and how they can access the presentation.</p>
<p>Go to www.menti.com and use the code 5478 1004</p> <p></p>  <p></p> <p>Criteria for success: Be able to write a linear function on the form $f(x) = mx + b$ if you know its two points.</p> <p>Slide 3 - Criteria for success.</p> <p>Introduce a 'criteria for success' for the lesson with a problem that will be revisited and solved at the end of the lesson. The idea here is to make the learning visible, so the students can see for themselves that they've actually learned something.</p>	<p>Go to www.menti.com and use the code 5478 1004</p> <p></p> <p>According to you, a graph is a...</p>  <p></p> <p>Slide 4 - According to the students, a graph is...</p> <p>Let the students individually write down how they conceive the concept of a graph. This can help clarify potential misconceptions.</p>
<p>Go to www.menti.com and use the code 5478 1004</p> <p></p> <p>Where would you place a banana?</p>  <p></p> <p>Slide 5 - Where would you like to place the banana?</p> <p>The students should realize that the banana should be placed in a specific position depending on the <i>relationship</i> between the y-axis and x-axis as a warm up.</p>	<p>Go to www.menti.com and use the code 5478 1004</p> <p></p> <p>Where would you place a watermelon?</p>  <p></p> <p>Slide 6 - Where would you like to place the watermelon?</p> <p>The students get another opportunity to practice the idea of placing the watermelon in a specific position depending on the <i>relationship</i> between the y-axis and x-axis.</p>

 <p>Look at the alternatives. Where is the treasure located?</p> <p>$f(x) = y$</p> <p>Press ENTER to draw correct. Press S to draw wrong.</p> <p>Results are hidden</p>	 <p>Look at the alternatives. Which coordinates represent the points A, B, and C?</p> <p>$f(x) = y$</p> <p>Press ENTER to draw correct. Press S to draw wrong.</p> <p>Results are hidden</p>
<p>Slide 7 - Where is the treasure located?</p> <p>The thought here is to get the 'aha' feeling in understanding the need of a uniform mathematical language. Specifically points in the x-y-coordinate system are given in the form (x,y) (not (y,x)).</p>	<p>The students get an opportunity to reinforce the understanding of the (x,y)-representation with points.</p>
 <p>A graph is used to visualize a function. You can think of it as the relationship between the function machine and the input-output pairs.</p>	 <p>By looking at the graph and the alternatives, what value is -4 and 2 being transformed to by the function-machine?</p> <p>$f(x) = y$</p> <p>Press ENTER to draw correct. Press S to draw wrong.</p> <p>Results are hidden</p>
<p>Slide 9 - Function machine that takes x:s and gives y:s.</p> <p>This is a key slide. The slide builds upon the previous module ('The concept of a function') and links the 'function-machine-methaphor' with the graphical representation.</p>	<p>Slide 10 - What value is -4 and 2 being transformed to by the function-machine?</p> <p>This is a key slide. The idea here is to strengthen the intuition from the previous slide, specifically that a function could be represented and visualized with a graph.</p>
 <p>The function of this machine is visualized in the graph. What is the function of the machine?</p> <p>$f(x) = y$</p> <p>Press ENTER to draw correct. Press S to draw wrong.</p> <p>Results are hidden</p>	 <p>Here is the graph of the same function as in the last slide $f(x) = 0,5x + 1$. When $x = 1$, what is the value of $f(x)$ (or y)?</p> <p>$f(x) = y$</p> <p>Press ENTER to draw correct. Press S to draw wrong.</p> <p>Results are hidden</p>
<p>Slide 11 - What is the function of the machine?</p> <p>This is a key slide. The idea here is to strengthen the two previous slides, specifically that a function could be represented and visualized with a graph.</p>	<p>Slide 12 - Take the previous function $f(x)=0,5x+1$ and insert $x=1$...</p> <p>The idea here is to reinforce the connection between a function and its graph. The idea here is to understand that a specific input (on the x-axis) for the function gives a specific output ($f(x)=y$ on the y-axis) and that this is valid for any x.</p>



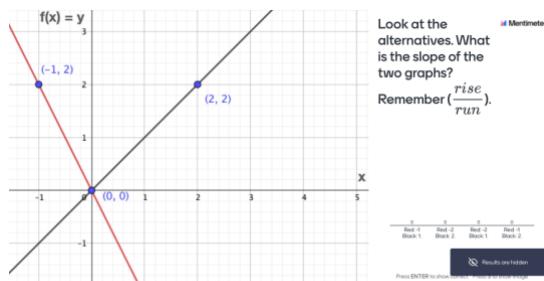
Slide 13 - The slope of a graph [Contentslide]

This is a key slide. The purpose is to understand that the m-value refers to the rate of change between x and y. The idea here is to gain a conceptual understanding rather than a procedural.



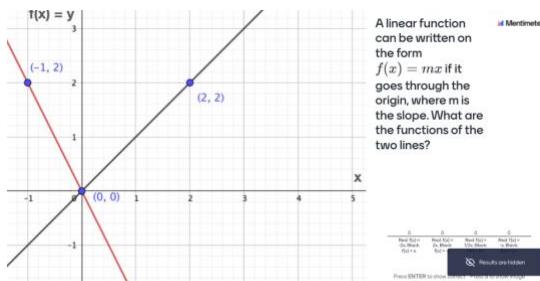
Slide 14 - What is the slope of this graph?

Here the (rise/run)-concept is reinforced to strengthen the understanding of the m-value.



Slide 15 - What is the slope of the two graphs?

Here we introduce the negative slope. The idea here is that the learner will gain insights about the challenges to apply the (rise/run)-concept to a negative slope.



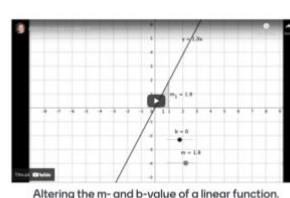
Slide 16 - What are the functions of the two lines?

By applying the (rise/run)-formula the student should be able to formulate the functions.

Any linear function can be written on the form $f(x) = mx + b$ where m is the slope and b is the y-intercept.

Slide 17 - The formal expression of the linear function. [Content slide]

'Introduction' to the formal definition of a linear function. Try to inform the students that the function is only depending on one variable (x) and that the m-value and the b-value are constants.



Slide 18 - Altering the m- and b-value of a linear function. [Content slide]

A video that shows what happens to a function in a visual representation when altering the constants m and b.

<p>Write the three y-intercepts for the graphs!</p> <p>Press S to show image</p> <p>Results are hidden</p> <p>Press ENTER to show results. Press Esc to close image.</p> <p>MENTIMER</p>	<p>For any x, which function has the highest value AND which of these three functions increases the most?</p> <p>Press ENTER to show results. Press Esc to close image.</p> <p>Results are hidden</p> <p>MENTIMER</p>
<p>Slide 19 - Write the three y-intercepts for the graphs.</p> <p>The idea here is to develop an understanding of the b-value or in other words the y-intercepts.</p>	<p>Slide 20 - For any x, which function has the highest value respectively increases the most?</p> <p>The purpose here is to give the student something to ponder with the intent of making them think about what <i>increase</i> really means in this context and idea of parallel lines.</p>
<p>Criteria for success revisited: Write the linear function f on the form $f(x) = mx + b$</p> <p>Press S to show image</p>	<p>Go to www.menti.com and use the code 5478 1004</p> <p>What is still unclear about the graph of a linear function?</p> <p>No questions from the audience!</p> <p>Incoming questions will show up here so that you can answer them one by one.</p> <p>MENTIMER</p>

Go to www.menti.com and use the code 5478 1004

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Quiz!

Results are hidden

Press S to show image

Press ENTER to show results. Press Esc to close image.

Slide 23-29 - Quiz Competition.

Ask the student to enter their real names. Let the students compete in a fun way while you as a teacher can get a sense of the individual level of knowledge of different students. This information could be used to give extra support when needed.

References:

Asiala, M., Cottrill, J., Dubinsky, E., & Schwingendorf, K. E. (1997). The development of students' graphical understanding of the derivative. *The Journal of Mathematical Behavior*, 16(4), 399-431.